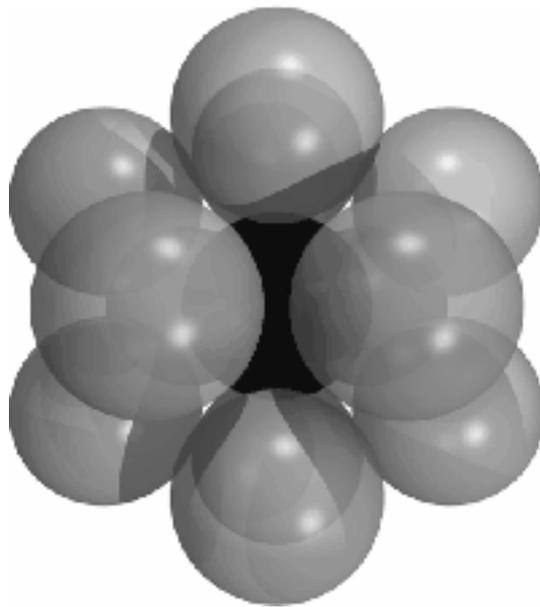


Maths for the Mystified

An exploration of the history of
mathematics and its relationship to
modern-day science and computing

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To Christine



"Kissing Numbers" graphic © Wolfram Inc. (Mathworld)

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Front and rear cover images: Sections of the Mandelbrot set, generated by the author. See further, Section 13.2, Recursion and chaos

Preface

The following is a true story...

Larry Walters, a delivery driver for a Hollywood film production company, had a boyhood dream to fly. As he sat in his backyard watching jets fly overhead he hatched his daring scheme. He purchased 45-50 weather balloons from an Army and Navy surplus store, tied most of them to a garden chair which he had tethered to his friend's car, and filled the balloons with helium gas. On June 2nd 1982 he strapped himself into the chair with some food, drinks, water containers to act as ballast and a pellet gun. He figured he would pop a few of the many balloons when it was time to descend.

Needless to say, Larry doesn't appear to have spent much time studying maths or science at school. Helium is lighter than air and will generate a lift of just under one ounce per cubic foot at sea level (or about 1 gram per litre if you prefer metric measurements). This doesn't sound much, but total lift depends on the number of balloons and their size. The balloons were quite large - around 3½-4 foot in radius (or 7-8 foot across, based on what Larry said at the time and from experiments with weights that he and his friends are reported to have conducted). Assuming a 4 foot radius each balloon would hold around 270 cubic feet of helium* and thus generate a lift of over 14lbs (6.4kgs). So 40 or more of these would lift a man, chair and ballast without any problems! But lift is reduced by around 7.5% per 1000 feet, depending on the variations of both pressure and temperature with altitude. So, luckily for Larry, sooner or later he would stop rising...

Larry's plan was to cut the tether lines and lazily float up to a height of about 30 feet above his back yard and maybe out across the nearby desert area towards the Mountains, enjoying a few hours of flight before coming back down. But things didn't work out quite as Larry planned... When his friends cut the nylon cords anchoring the chair to the car he did not float lazily up to 30 feet. Instead, he streaked into the Los Angeles sky as if shot from a cannon, pulled aloft by the powerful lift of the helium balloons. He didn't level off at 100 feet, nor did he level off at 1000 feet. After climbing and climbing he levelled off at 16,000 feet. At that height he felt he couldn't risk shooting any of the balloons lest he unbalance the load and really find himself in trouble. So he stayed there, drifting, very cold and frightened, for more than 2½ hours.

Then his path crossed the primary approach corridor of Los Angeles International Airport. A TWA pilot first spotted Larry. He radioed the control tower and described passing a guy in a garden chair... with a gun... at 16,000 feet! After a period of some disbelief radar confirmed the existence of an unidentified object floating high above the airport. Eventually Larry gathered the nerve to shoot a few balloons before accidentally dropping the gun, but fortunately by then he had done enough and slowly descended. As he approached the ground the hanging tethers tangled and caught in a power line, resulting in the local electricity authority having to cut off the supply from a Long Beach neighbourhood while Larry was rescued. He eventually managed to climb down to safety where waiting members of the Los Angeles Police Department arrested him. As he was led away a reporter dispatched to cover the daring flight asked him why he had done it. Larry replied nonchalantly, "A man can't just sit around."

Report developed from contemporary newspaper reports and material published on the Darwin Awards website (www.darwinawards.com) and Mark Barry's more complete "Lawn Chair Pilot" website (<http://www.markbarry.com>)

Corrections to published material kindly provided by Mark Barry (personal communication). Mark has interviewed several of those present, excluding Larry Walters himself as sadly Larry later took his own life. Photos and an audio track covering the flight are included on the Lawn Chair Pilot web site. The photo here shows Larry shortly after takeoff, having just lost his glasses!

A small number of enthusiasts continue to fly 'cluster balloons' like Larry. See <http://www.clusterballoon.org/> for more details

* see Appendix 1 for details of this calculation



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Structure of the book

This is a book for people who want (or need) to analyse practical real-world problems, but are unsure how to proceed with confidence. In an effort to dispel such fears and difficulties I examine and solve a wide variety of real-world problems through a consideration of the processes of measurement and the application of very simple computational methods. This approach enables us to examine, illustrate, test and develop a broad range of ideas and procedures that work, from the comfort of our own 'garden chair', without recourse to complicated equations or programming.

Throughout this book I make use of modern desktop computer systems and the mathematical software that has been developed for them, treating these as a special kind of experimental laboratory. The computer as laboratory enables a quantum leap in our capabilities as number crunchers, professional problem solvers or simply interested amateurs. This is in part a result of the speed of computers, but increasingly as a result of their incorporation of past knowledge and experience - the accumulated results, findings and experience of generations of mathematicians. This experience is stored for our quick reference, embodied in procedures that ensure mistakes are minimised and that 'correct' processes are followed, and then presented to us through interfaces (input and output) that are understandable and visually stimulating. Such facilities do not remove or even reduce the need for understanding and insight - to an extent they demand an even greater level of understanding and care - but they do enable the process of investigation to be far speedier and often to be more effective, rewarding and fun.

In order to explore these issues I have divided this book into three main parts:

Part I deals with the why? why do we mainly use a number system based on 10 elements? why do we have numbers like zero (0) and minus 1 (-1)? why don't we (generally) have a symbol for 10, unlike the Greeks and Romans? why do we have different groups of numbers with strange names like Integers, Imaginaries and Irrationals? and why does mathematics seem to be full of strange symbols and equations? Answering these questions involves delving into some of the history and personalities involved in the development of this subject. It also leads us to look at the history of measurement (of lengths, weights, volumes, times, etc.) and the notions of dimension and connectivity. These are fascinating stories in their own right, but also highlight the role of experimental data in formulating our understanding of numbers and the central role they play in practical and theoretical analysis.

Part II examines the process of discovery through experimentation, focusing on the relatively new world of the computer as laboratory. Computers, and their predecessors electronic and mechanical calculators, and before these vast teams of human 'computers', have long been used to carry out tedious and repetitive numerical processing tasks. But it is a very recent phenomenon that has seen their function developing towards an experimental, interactive, enquiry-led mode of usage. This extends their application beyond that of computational and educational tool, to one in which the tool becomes a key part of the process of discovery and refinement. But our new, computationally rich world, introduces important technical issues that are unfamiliar to most people (except for professional software engineers and some mathematicians). I attempt to address many of these issues, illustrating the materials with a wide range of examples and available software tools. The software tools utilised and discussed include: Microsoft's Excel spreadsheet product; Maplesoft's Maple package, which is particularly good at *symbolic* processing (working with symbols and expressions as well as numbers); and Waterloo Software's MATLAB package (MATrix Laboratory), which is a general purpose scientific suite of software with strong support for manipulation of blocks of numbers and expressions. Student versions of Maple and MATLAB are available at much reduced prices. Finally, use is made of several innovative online (web-accessible) tools, which are described and discussed in the text. For all of us wishing to use computers the first step is to be able to enter numbers and expressions, obtain meaningful output, and to incorporate these results into documents. The latter requirement presents a number of difficulties, so I also have included a discussion of some of these issues at the end of **Part II**.

Part III of this book provides a more detailed look at a selection of numbers and number sequences that are widely regarded as 'interesting'. Some argue that all numbers are interesting, and if a number appears uninteresting it is simply our failure to discover its possibilities. In fact there is a simple, but rather weak 'proof' that there are no uninteresting numbers. However, it is not practical to examine every possible number, nor even a large selection, so I focus on a personally

selected subset. Many interesting numbers have even more interesting stories associated with them, and some of these are provided in the final Part of the book.

A selection of suggested books to read, web sites to explore and software to use are provided in Appendix 2 at the end of this book. In addition, on the author's web site (www.mdesmith.com) you will find copies of spreadsheets, program extracts, useful web links, additional materials and sample problems.

Intended Audience

I have written this book to be of help and interest to a wide range of readers: for those interested in learning a little more about numbers, mathematics and computer tools in general; for those who are simply seeking a refresher course with a modern perspective; for those who find mathematics and computing a rather daunting but necessary part of the modern world that they wish to understand more thoroughly; and for those interested in using computers as tools of discovery - examining, testing and visualising information and ideas in new ways. To this extent the text may serve as introductory reading for students (and hence as a resource for teachers and lecturers) including those in higher and further education (colleges, universities). It is intended to be of particular value to those studying subjects in the social sciences, physical and earth sciences, and life sciences. Such disciplines demand a thorough appreciation of mathematical and statistical ideas but incorporate very little pure mathematics or software engineering in their coursework.

Although I discuss and make use of some simple mathematics, this is not a maths text and does not assume a detailed knowledge of this subject. Readers who are more confident in some of these areas may find that they can skim through several of the Sections, in particular those dealing with the different groups of numbers that we commonly encounter. Frequent use is made of software tools, but once again little or no knowledge of programming is required. I do, however, explain the principal forms of notation and conventions used in mathematics, and some of those that apply within computing. It is useful to understand where this notation comes from and how it is used in order to be able to read and appreciate these subjects more fully. Thus, in the process of exploring numbers and data, I cover a wide range of topics including many important mathematical concepts - from number theory to topology, and from basic statistics to calculus, but introduced in an accessible and largely non-traditional manner.

This book also explores a wide range of application areas, from understanding gravity to computing the speed of light, from Internet security to digital audio and image processing, and from the exploration of number series to better ways of obtaining arithmetic results. My hope is that readers will find the material both interesting and enjoyable, and will become confident enough as a result to read more widely in this field, to conduct their own experiments with numbers and to explore the wonderful array of resources now available on the Internet. Amongst the latter is the MacTutor web site of University of St Andrews. This site provides bibliographic details of many famous mathematicians and brief discussions on key topics in mathematics. With their kind permission I have used some of their excellent material in the provision of brief profiles of several key figures from the history of this subject.

When reading this book it is useful to have a pencil and paper to hand, and if possible, have access to a desktop computer to reproduce and test out many of the examples. I make frequent use of Microsoft Excel as a form of sophisticated calculator, so limited familiarity with Excel is helpful but not a pre-requisite. An essential component of this book is the use of many worked examples, including those involving measurement of the physical world. If in doubt (or even if not) I strongly recommend that readers try reproducing the numerical examples, looking for any errors, weaknesses or unstated assumptions.